One Approach to Determine the Geoid Local Trend at Montevideo Area

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Abstract. In the decade of the 60´s, one specific geodetic system was calculated and established for Montevideo, the capital of Uruguay.

The Uruguayan Geodetic Network and its height system ROUUSAMS was finished in its high precision level in 1961. In 1965, the adjustment of the network was performed, and the Yacaré Datum (International Ellipsoid) was defined, being the base of the mapping system until today.

The generalisation of the use of the GPS and its application in levelling increases the needs of the good knowledge of the geoid and the determination of different heights. Taking in consideration that Montevideo has its own Reference System (the Consejo Departamental de Montevideo - CDM) the study of the geoid in this particular area will be performed and supported by the Universidad de la Republica (UDELAR) in agreement with Intendencia Municipal de Montevideo (IMM).

This paper shows the trends of the geoid in Montevideo, the processing of data collected from GPS and the levelling network including a refined treatment of the data collected through statistical tests and robust estimators.

Keywords. Geoid determination, levelling network, Montevideo CDM system, global geoid models, orthometric height derivation, robust estimators.

1 Introduction

The geoid is defined as the gravity equipotential surface which best approximates the mean sea level (MSL) over the entire Earth. It has been defined as the datum for the orthometric height system.

The irregular shape of the geoid, however, does not allow an easy computation of the horizontal positions of points. Therefore, a reference surface of regular shape, usually a biaxial ellipsoid, is selected to best approximate the geoid either locally or globally.

The geometric relationship between the geoid and the reference ellipsoid surface can be fully described by their separation (N) and the slope of the geoid with respect to the reference ellipsoid (Ø). The geoid undulation (N) refers to the separation between the reference ellipsoid and the geoid measured along the ellipsoidal normal, whereas the height anomaly refers to the separation between the reference ellipsoid and the quasigeoid, also measured along the ellipsoidal normal.

Surveying often involves the determination of heights of points; such heights are related to the vertical datums, which in this sense are in fact reference surfaces for heights. Height datums may be as simple as arbitrary datums , where heights of points in small survey areas are related to fixed points (benchmarks) with arbitrarily assigned heights, or as complex as national height datums, ROUUSAMS, where heights are related to an approximation of MSL (Mean Sea Level) around the coastline of Uruguay.

The determination of heights is closely allied to engineering works concerned with the control and flow of water. Water obeys the laws of physics and flows “downhill” from one equipotential surface to another, in fact a “free” body of water will form its own equipotential surface. This natural occurrence is the reason why an equipotential surface is the most sensible datum for heights. It also provides the reason why most countries adopt some form of MSL as the datum for heights, since all waters of the Earth discharge to the ocean.

Estimation of orthometric heights from GPS measurements requires a knowledge of the value of separation (N). This value cannot be determined exactly, but instead must be interpolated from pre-prepared geoid models, or computed from geopotential models (EGM96, OSU91). That means the possibility of rapid height determination using

GPS. The agreement between the UDELAR and IMM covers both aspects, the scientific research and the engineering application in public works.

2 Description of the Project

Global Positioning System (GPS) observations were made in a test area of Montevideo for determining the trend of the geoid. The network that was measured by 4 GPS receivers comprised 51 stations (1 SIRGAS reference station included), covering approximately 20 km * 20 km. All the points taken, belong to the CDM levelling system, being points of first and second order.

Two different geoids (OSU91 and EGM96) were used to convert the GPS ellipsoidal heights to orthometric heights. The GPS levelling accuracy was investigated by deriving an error model with the aid of the computed height differences between the GPS-levelled heights and the spirit-levelled heights. A statistical study of the data was performed, and several plots are analysed including 3D perspective views.

The general purpose of this work, is to describe and quantify the general trend of the geoid over Montevideo. This trend can be expressed by the value of the correction to be applied to the height modelled and calculated from the GPS observations. The iso-values have been drawn on a digital map supporting the same geodetic reference system (CDM). Then it is possible to calculate an adequate grid to find the correction or the interpolated value. Before that, many statistical process were applied to verify the consistence of the data observed (GPS) and the data collected (levelling network data)

Ellipsoidal heights (h) resulting from GPS observations and orthometric heights (H) which are reckoned from the geoid, are related to each other according to the following simple equation: $H=h-N$. The simple equation above represents the principle of the method referred to as GPS levelling, which states that if N at the observation site is known, and h is determined by GPS observations, then H values of the site can be computed. The relationship between h and H also states that the accuracy of H obtained with GPS is dependent on the accuracy of the GPS determinations and the accuracy of N.

3 Collecting and Processing of the Data

One of the main problems involved in this study is to know the state of the data. If the data concerning the level of the points has a mistake or relevant errors, then the following computations will be affected by the error propagation. We can identify two different groups of data:

Data observed $\Rightarrow$ GPS measurements.
Data "captured" $\Rightarrow$ Levelling network.

The reliability of the GPS observations can be controlled by several field methods and the software used in the processing. But the control of the data captured from the existing levelling network implies to develop a specific methodology based in statistical testing and robust estimators.

3.1 Detection of Outlier or Blunder Errors in the Levelling Data

We classify the errors in random errors, systematic errors and outlier or blunder errors. Optimum estimators and diverse techniques of observation and adjustment are thought and designed under the assumption that the observations have exclusively random or stochastic errors.

Mathematically, the systematic errors are seen as different types of divert making the expectation value of the error different from zero. In the geodetic and topographic measuring fields the systematic errors normally come from bad calibration of the used instruments, environmental effects, personal equations of the observer, etc. The most reliable way to detect the existence of systematic errors is through statistical tests and the application of robust estimators .

We mathematically understand by blunder errors those random errors that are 3 to 20 times bigger than the expected standard errors according to pre-established accuracy and tolerance. Any form of preventing the error occurrence results insufficient to dismiss the existence of blunder errors specially in samples of big amount of observations. One of the most important problems in the probabilistic and statistic theory is the later detection of blunder errors in the observational work through the analysis of residuals.

The modelling of the observational errors helps us to set up the procedure rules for the detection of non-random errors:

1. Starting from the premise that the observational errors are random errors with a specific associated distribution; then this is the hypothesis $H_{0.}$ The alternative hypothesis H_1 is that the error will not be random according to an associated distribution.
2. We can build derivative variables with the same distribution according to the previous condition.
3. Finally, we must test the values taken from the previous statistics (samples) against the critical theoretic values for a certain level of significance or "risk" α.

If the hypothesis testing is accepted we can definitely take in consideration the point 1. Otherwise, we must suspect the existence of blunders in the observational data set. This testing can be directly applied on the observation or after a preliminary adjustment by least squares.

3.2 Tests for the Detection of Blunders

Before stating when the errors coming from measurements have random performances or not we must know the properties of random errors.

- The arithmetic mean ε_i must be approximate to zero when the number of observations (n) is big enough.
- Positive or negative sign errors have the same chance of occurrence.
- Short magnitude errors have a bigger probability of occurrence than the absolute great magnitude errors.
- Under certain measurement conditions, the absolute magnitude of the errors must be within certain limits.

Taking the properties mentioned above in consideration we can build many statistical tests whether they are random errors or not. Here we state in a summarised way some five different kinds of tests being the last ones among the most important ones, as it allows to find a mistake or blunder error: Test the number of positive errors against negative ones, test the order of negative errors against positive ones, test the sum of squares of the negative and positive errors, test the sum of errors, test the maximum absolute value of the errors.

3.3 Robust Estimators

Robust estimators are those ones that become insensitive to the limited variations of the distribution functions, for example, in case of blunder errors occurrence. These types of estimators are based on other models or techniques different from the concept of the least squares. This topic has become crucial in the whole area of studies in the quality control of geographic data and this is showed by a variety and increase in the articles published nowadays about the topic.

3.4 Regression Diagnostics

One of the robust estimators applied in this study, is the “regression diagnostics”.

The regression diagnostics supposes a preliminary adjustment through least squares and is processed after the following:

- Initially an adjustment of the whole data is performed through least squares method.
- The residual is computed for each observation.
- All the observations that do not fulfil the minor conditions will be defeated.
- A new adjustment is performed with the remaining observations.

The success of this process depends on the quality of the initial adjustment, and does not guarantee a correct result, but as well as the GRIT (see below) this process works very well with a moderate percentage of blunder errors.

3.5 Great Residuals Iterative Test (G.R.I.T.)

The robust estimator “regression diagnostics” and a variation of the previous technique is the G.R.I.T. (F.Barbato, 1998). It is highly efficient in data sets of $n \approx 30$ as a first order approximation.

The main idea of this test from the residuals calculation [$V_i = L_i - \overline{X}$] is to arrange them from greater to minor through absolute magnitude order. This model presupposes the existence of fewer blunder errors.($\cong< 3\%$).

Before determining the residuals it is necessary to identify and immediately eliminate mistakes that can involve, for example, the variation of a major or minor order of the power in 10^i of the measures. This will help that the initial mean calculation will not look seriously distorted.

The greatest V_f (could be more than one) is taken, and it is important to control that its residual does not exceed a certain tolerance value (ψ_f) according to the pre-established conditions, methodology, instruments, etc. In our case, where the test is carried out taking the “differences” between heights determined in the field and those computed through geopotential models as random variables, ψ_f will be defined as a function of the combined accuracies from the estimated level accuracy, geoidal accuracy, GPS accuracy and other components. The corresponding measurement to that residual is eliminated from the data set, and the residuals with the new measures are re-calculated, keeping the statement of tolerance. But for the consecutive cases a smoothing rate between 10% to 20% is established. This is pointing out that we must make the best use of the limited quality of observations, leaving for a next stage the determination of the set consistence with the normal distribution.

We will classify as a “new suspect” of blunder error, that residual which most strays from [$1.20*\psi_f$]. The (1.20) factor has the aim to create a smoothing interval to make the adoption to samples of $n<30$ possible, where the elimination of observations can degrade the density function [f(x)] and associated distribution properties. It has been experimentally proved that eliminating the biggest blunder error, the model gets “extremely severe” with the reminding “outliers”.

This iteration is carried out until the quantity of blunder errors will not exceed the pre-established limit, which means that the blunder errors would be related to “abnormal” variations or disorder in the measuring process, being necessary to check and start again with the measuring process.

Summarising, the GRIT estimator suggests:

1. [$V_i = L_i - \overline{X}$] ordered from major to minor
2. discarding of measures different in $>10^i$ (i >=1)
3. determination of ψ_f
4. rejection of V_f measures that do not fulfil the condition
5. re-calculation of the values

To complete the procedure, after refinements accepted by the GRIT process, we are ready to go on with the “verification of systematics” tests and the distribution control associated to the “edge” blunder errors whose determination is not clear or definitive.

Mainly, all these procedures have been applied to the differences between CDM level and heights computed from the EGM96 and OSU91 models. These values can be named also as “N computed” in contrast to the “N observed”. From these five tests a sample rejection by only one of the techniques will be enough to make us check the measurement values.

3.6 Results of the Test

Applying the methodology developed before we found four points suspected to be affected by gross errors: Points ID: 10393, 10396, 407B, 604C.

Analysing the data, and re-occupying the four “problematic” points with GPS we conclude that two of these were wrongly computed (10396 and 604C) and the rest (10393 and 407B) have a mistake in the original information from the levelling network. Then the original sample of 51 points was reduced to 49.

After that, all the data was computed again, resulting in more accuracy obtained from the statistical analysis.

4 Results

In tables 1 and 2 we show the results after the deputation of the sample:

Table 1.Statistical parameters of the raw data.

STATS	N_EGM96	N_OSU91	DIF(Ns)	Niv	h-NivCDM	H96-NivCDM	H91-NivCDM
Mean	14.34	15.32	1.04		14.92	0.62	-0.41
Maximum	15.00	16.00	1.40	134,820	16.60	2.43	1.36
Minimum	13.90	15.00	0.53	3,800	14.31	-0.06	-1.10
Range	1.10	1.00	0.87	131,020	2.29	2.49	2.46
Variance	0.17	0.03	0.03		0.17	0.18	0.17
Std.Deviation	0.41	0.18	0.17		0.41	0.42	0.41

Table 2 .Statistical parameters after the testing process.

STATS	N_EGM96	N_OSU91	DIF(Ns)	Niv	h-Niv	H96-Niv	H91-Niv
Mean	14.35	15.32	1.03		14.84	0.54	-0,51
Maximum	15.00	16.00	1.40	134.820	15.25	0.84	-0,08
Minimum	13.90	15.00	0.53	3.800	14.39	0.24	-0,93
Range	1.10	1.00	0.87	131.020	0.86	0.60	0,85
Variance	0.17	0.03	0.03		0.04	0.02	0,02
Std.Deviation	0.42	0.18	0.17		0.19	0.14	0,14

Comparing the two tables, we can see the improvement of the main statistical parameters after the testing. The adequate geographic distribution of the GPS points is shown in figure 1.

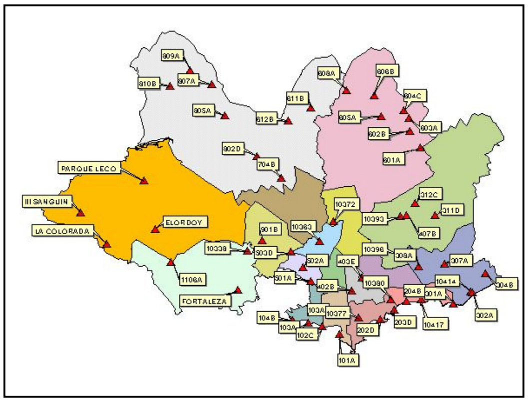

Fig.1 Geographic Distribution of the points.

The four “problematic” points are located on the central and east part of Montevideo, inside one radius of 500 m.

Figures 2 and 3 illustrate the trend of the geoid over the Montevideo area depending on the global geoid model used for the height determinations.

To give an idea about the geomorphologic structure of Montevideo, figure 4 shows the DEM (Digital Elevation Model) computed from the contour level spacing two meters interval.

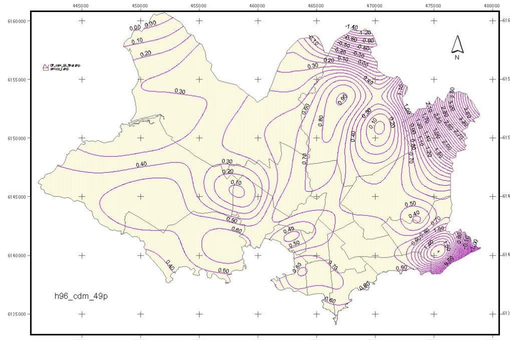

Fig.2 Contour computed from the difference EGM96 - Niv.

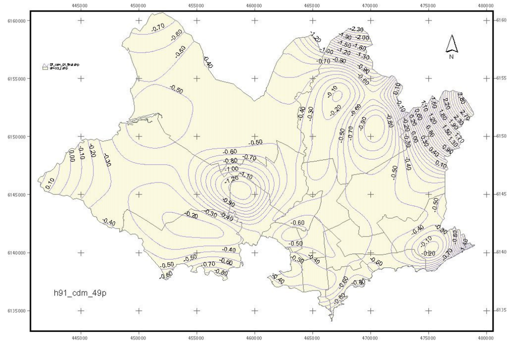

Fig.3 Contour computed from the Dif_OSU91_Niv.

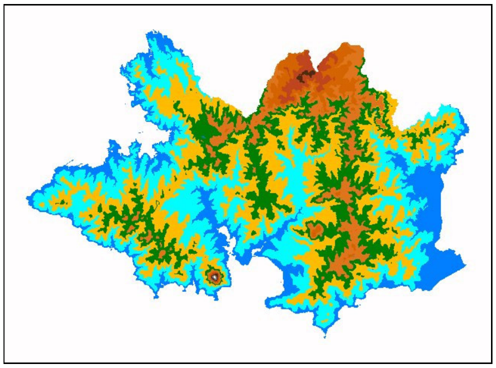

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Fig.4 Digital Elevation Model of Montevideo.

5 Discussion of Accuracy

Height differences between GPS levelling and spirit levelling results contain errors originating from three main sources: GPS determinations, spirit levelling and geoid heights. The accuracy of the computed height differences may be derived from the accuracy of their main components. If the GPS accuracy is denoted by σ_h, levelling accuracy by σ_H, accuracy of geoid height differences by σ_N, and the accuracy of the MSL determination (this accuracy affects to the surface reference of levelling) by σ_U, the accuracy of the differences between GPS levelling and spirit levelling may be computed from

$$\sigma^2_{H(GPS-lev)} = \sigma_h^{\,2} + \sigma_H^{\,2} + \sigma_N^{\,2} + \sigma_U^{\,2} \quad (1)$$

σ_h: Estimation of the formal accuracy of GPS heights is difficult due to the optimistic standard deviations resulting from the GPS software. Therefore we need to obtain more realistic estimations correcting the value σ_h by a factor.

σ_H: Adjustment of the precise levelling loops for lower order results in $6 \text{ mm} \sqrt{d(km)}$.

σ_N: The accuracy of geoid height differences can be estimated by the rule of thumb derived by Vermeer (1995).

σ_U: Taking into account that our vertical datum has systematic errors originated in the location of the zero tide gauge (mixed fluvial and oceanic regimen), we can approach σ_U to ± 0.10m.

The estimated GPS levelling error is then

$\sigma_{H(GPS-lev)} \cong 0.17$ m.

6 Conclusions

This project represents the initial study of the geoid in Montevideo under the cooperation between the Universidad de la Republica and Intendencia Municipal de Montevideo. The raw collected data and its processed results require a deep study of the local levelling network in this area.

The initial statistical parameters show strong differences in the max. and min. values, not acceptable for height computations. A testing process was applied to detect gross errors and to improve the quality of the data. It is necessary to increase the number of GPS points and test some regions of the levelling network.

The first technical conclusion is that the results derived from EGM96 are better than the OSU91 model. This can be concluded analysing the data from table 2.

The second one is that for engineering surveys limited to a relatively small area where a sufficient number of geoid heights are known from a combined GPS and geodetic levelling technique, one grid with interpolated values can be supplied in the near future.

The last one is the need to review the original data from the levelling network. Considering that we found many gross errors in the data (≅ 4%), this technique may be a fast testing procedure to evaluate the levelling network of Montevideo.

Finally, we mean that it is clear to continue this project oriented to satisfy the needs of a lot of GPS users, that means, to improve the quality and performance of the GPS levelling in Montevideo.

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